Solving Intuitionistic Fuzzy Linear Fractional Programming Problem

M. Jayalakshmi
Assistant Professor, Department of Mathematics,
School of Advanced Sciences, VIT University,
Vellore, India

Abstract: A new method is proposed to find an optimal intuitionistic fuzzy solution to fully intuitionistic fuzzy linear fractional programming (IFLFP) problem, in which ranking functions are not used. Here, the IFLFP problem is transformed into an equivalent crisp linear fractional programming problem which is then solved by linear programming (LP) technique and / or linear fractional programming technique. The procedures for the proposed methods are illustrated with the numerical examples.

Keywords: Triangular intuitionistic fuzzy number, linear programming problem, linear fractional programming problem, intuitionistic fuzzy linear fractional programming problem.

I. INTRODUCTION

Linear fractional programming (LFP) problems are a special type of nonlinear programming problems in which the objective function is a ratio of linear functions and the constraints are linear functions. In real life situations, linear fractional models arise in decision making such as production planning, financial and corporate planning, health care and hospital planning. In the literature, several methods [1, 3, 5, 8, 10] have been suggested to solve LFP problems.

In fuzzy decision making problems, the idea of maximizing decision was anticipated by Bellman and Zadeh [4]. In the literature, many researchers have developed various algorithms to solve fuzzy linear fractional programming (FLFP) problem. Recently, Nachammai et al. [7] considered FLFP problem by using ranking method based on metric distance. Pop & Stancu Minasian [9] and Bogdana Stanojevi’ca & Stancu-Minasian [11] used deterministic multiple objective linear programming problem by quadratic constraints to work out FLFP problems. Pandian and Jayalakshmi [8] have proposed a new method namely denominator objective restriction method for finding an optimal solution to LFP problems by constructing two LP problems and also in the same paper they developed a new method namely, decomposition-restriction method to solve FLFP problem based on the above said method, without using fuzzy ranking function.

The Fuzzy set was extended to develop the Intuitionistic fuzzy set, introduced by Attanosov [2], by adding an additional non-membership degree and hesitancy degree. Based on the literature study, it was found that there is a little work carried out to solve IFLFP problems. More recently, Sujet Kumar Singh and Shiv Prasad Yadav [12] have solved IFLFP problem by transformed into an equivalent crisp multi-objective LFP problem which is then reduced into a single objective LP problem.

II. PRELIMINARIES

We need the following mathematical orientated definitions of IF set, triangular IF number and membership function and non-membership function of an IF set/number which can be found [6].

**DEFINITION 1** Let X denote a universe of discourse and \( A \subseteq X \). Then, an IF set of A in X, \( \tilde{A} \) is defined as follows: \( \tilde{A} = \{ (x, \mu_{\tilde{A}}(x), \vartheta_{\tilde{A}}(x)) : x \in X \} \)

Where \( (\mu_{\tilde{A}}(x), \vartheta_{\tilde{A}}(x)) : X \rightarrow [0, 1] \) are functions such that \( 0 \leq \mu_{\tilde{A}}(x) + \vartheta_{\tilde{A}}(x) \leq 1 \), for all \( x \in X \). For each \( x \) in \( X \), \( \mu_{\tilde{A}}(x) \) and \( \vartheta_{\tilde{A}}(x) \) represent the degree of membership and non-membership values of \( x \) in the set \( A \subseteq X \).

**DEFINITION 2** A fuzzy number \( \tilde{a} \) is a triangular IF number denoted by \( (a_1, a_2, a_3)(\tilde{a}_1, \tilde{a}_2, \tilde{a}_3) \) where \( a_1, a_2, a_3, a_4 \) and \( a_5 \) are real numbers such that \( a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5 \) and its membership function \( \mu_{\tilde{A}}(x) \) and non-membership function \( \vartheta_{\tilde{A}}(x) \) are given below:
\[
\mu_a(x) = \begin{cases} 
\frac{x-a_2}{a_2-a_1}, & a_2 \leq x \leq a_3 \\
\frac{a_3-x}{a_3-a_2}, & a_3 \leq x \leq a_4 \\
0, & \text{otherwise}
\end{cases}
\]

\[
\theta_a(x) = \begin{cases} 
\frac{a_3-x}{a_3-a_2}, & a_1 \leq x \leq a_2 \\
\frac{a_2-x}{a_2-a_1}, & a_2 \leq x \leq a_3 \\
0, & \text{otherwise}
\end{cases}
\]

Let IF(R) be a set of all triangular IF numbers over \(R\), a set of real numbers. Based on ordering relation in interval theory/fuzzy set theory, we define the following:

**Definition 3** Let \(\tilde{a}^i = (a_2, a_3, a_4)\) and \(\tilde{b}^i = (b_2, b_3, b_4)\) be in IF(R). Then,

(a) \(\tilde{a}^i + \tilde{b}^i = [(a_2 + b_2, a_3 + b_3, a_4 + b_4) + (a_1 + b_1, a_2 + b_2, a_3 + b_3)]\)

(b) \(\tilde{a}^i - \tilde{b}^i = [(a_2 - b_2, a_3 - b_3, a_4 - b_4) - (a_1 - b_1, a_2 - b_2, a_3 - b_3)]\)

(c) \(\tilde{a}^i \odot \tilde{b}^i = [(a_2 b_2, a_3 b_3, a_4 b_4) + (a_1 b_1, a_2 b_2, a_3 b_3)]\)

(d) \(k(a_2, a_3, a_4) = (a_1, a_3, a_3)\)

**Definition 4** Let \(\tilde{a}^i = (a_2, a_3, a_4)\) and \(\tilde{b}^i = (b_2, b_3, b_4)\) be in IF(R). Then,

(a) \(\tilde{a}^i\) is said to be lesser than or equal \(\tilde{b}^i\) if \(a_i \leq b_i\), \(i = 1, 2, 3, 4, 5\);

(b) \(\tilde{a}^i\) is said to be greater than or equal \(\tilde{b}^i\) if \(a_i \geq b_i\), \(i = 1, 2, 3, 4, 5\);

(c) \(\tilde{a}^i\) is said to be equal \(\tilde{b}^i\) if \(a_i = b_i\), \(i = 1, 2, 3, 4, 5\).

III. INTUITIONISTIC FUZZY LINEAR FRACTIONAL PROGRAMMING PROBLEM

Consider the following IFLFP problems having \(m\) IF constraints and \(n\) IF variables:

\[
\text{(P)} \quad \text{Maximize } \tilde{Z}^i \approx \tilde{\alpha}^T \tilde{x} \odot \tilde{a} \quad \text{subject to } \tilde{A} \otimes \tilde{x} \leq \tilde{b},
\]

where

\[
\tilde{x} = \left( x_1^1, x_1^2, x_1^3, \ldots, x_1^m \right), \quad \tilde{a} = \left( a_1^1, a_1^2, a_1^3, \ldots, a_1^m \right),
\]

\[
\tilde{A} = \left( A_{ij} \right)_{m \times n}.
\]

**Conclusions** Constraints in the decomposition problem in which at least one decision variable of the (P3) occurs and all decision variables are non-negative.
in (P3); all variables in the constraints and objective function in (P4) must satisfy the fuzzy triangular intuitionistic bounded constraints; replacing all values of the decision variables which are obtained in (P3) and all decision variables are non-negative;

Where $Z_3^*$ is the optimal objective value of the problem (P3);

(P2) :  Maximize $Z_2 = \sum_{j=1}^{n} \sum \\text{second value of } \left[ \frac{1}{m} \sum_{i=1}^{m} \left( c_j \left( x_i^1, x_i^2, x_i^3 \right) \otimes (x_i^1, x_i^2, x_i^3) \right) \right] \otimes \left( a_j, a_j, a_j \right)$

subject to

$Z_2 \leq Z_3^*$

Constraints in the decomposition problem in which at least one decision variable of the (P2) occurs and are not used in (P3) and (P4); all variables in the constraints and objective function in (P2) must satisfy the fuzzy triangular intuitionistic bounded constraints; replacing all values of the decision variables which are obtained in (P3) and (P4), all decision variables are non-negative; Where $Z_4^*$ is the optimal objective value of the problem (P3);

(P5) : Maximize $Z_5 = \sum_{j=1}^{n} \sum \\text{fifth value of } \left[ \frac{1}{m} \sum_{i=1}^{m} \left( c_j \left( x_i^1, x_i^2, x_i^3 \right) \otimes (x_i^1, x_i^2, x_i^3) \right) \right] \otimes \left( a_j, a_j, a_j \right)$

subject to

$Z_5 \geq Z_4^*$

Constraints in the decomposition problem in which at least one decision variable of the (P5) occurs and are not used in (P2), (P3) and (P4); all variables in the constraints and objective function in (P5) must satisfy the fuzzy triangular intuitionistic bounded constraints; all values of the decision variables which are in (P2), (P3) and (P4), all decision variables are non-negative; where $Z_4^*$ is the optimal objective value of the problem (P4).

(P1) : Maximize $Z_1 = \sum_{j=1}^{n} \sum \\text{first value of } \left[ \frac{1}{m} \sum_{i=1}^{m} \left( c_j \left( x_i^1, x_i^2, x_i^3 \right) \otimes (x_i^1, x_i^2, x_i^3) \right) \right] \otimes \left( a_j, a_j, a_j \right)$

subject to

$Z_1 \leq Z_2^*$

Constraints in the decomposition problem in which at least one decision variable of the (P1) occurs and are not used in (P2), (P3), (P4) and (P5); all variables in the constraints and objective function in (P1) must satisfy the fuzzy triangular intuitionistic bounded constraints; replacing all values of the decision variables which are obtained in (P2), (P3), (P4) and (P5), all decision variables are non-negative; where $Z_2^*$ is the optimal objective value of the problem (P2).

Now, we propose a new algorithm for solving fully IFLFP. The proposed method proceeds as follows.

STEP 1: Construct (P3), (P4) (P2), (P5) and (P1) problems from the given the fully IFLFP.

STEP 2: Using denominator objective restriction method and / or existing LFP technique, solve the problem (P3), then the problems (P4) and (P2), then the problems (P5) and (P1) in the order only and obtain the values of all real decision variables $x_j^3, x_j^4, x_j^5, x_j^6$ and $x_j^1$ for $j=1, 2, ..., n$ and the values of all objectives $Z_3, Z_2, Z_4, Z_5$ and $Z_3^*$. Let the decision variables be $x_j^3, x_j^4, x_j^5, x_j^6$ and $x_j^1$ for $j=1, 2, ..., n$ and the objective values be $Z_3, Z_2, Z_4, Z_5, Z_3^*$ and $Z_2^*$. Now, the proposed method for solving fully IFLFP is illustrated by the following numerical examples.

EXAMPLE 1 Consider the following IFLFP problem:

Maximize $Z_5 = (2, 4, 7) \otimes \hat{x}_1^f \otimes (2, 3, 4) (l, 3, 5) \otimes \hat{x}_2^f \otimes (l, 2, 4) (0, 2, 5) (l, 2, 3) (l, 2, 4) \otimes \hat{x}_3^f \otimes (3, 5, 8) (2, 5, 9) \otimes \hat{x}_4^f \otimes (1, 1, 2) (0, 1, 3)$

Subject to

$(0, 1, 2) (0, 1, 3) \otimes \hat{x}_1^f \otimes (0, 1, 2, 3) (l, 2, 4) \otimes \hat{x}_2^f \otimes (1, 1, 10, 27) (1, 10, 28) (l, 2, 3) (l, 2, 4) \otimes \hat{x}_3^f \otimes (0, 1, 2) (0, 1, 3) \otimes \hat{x}_4^f \otimes (2, 11, 28) (l, 11, 38)$

$\hat{x}_1^f, \hat{x}_2^f \succeq \hat{0}^f$.

Let

$\hat{x}_1^f = (x_1, x_2, x_3, x_4) (x_1, x_2, x_3, x_4)$, $\hat{x}_2^f = (y_2, y_3, y_4) (y_1, y_2, y_3, y_5)$ and $\hat{x}_3^f = (Z_2, Z_3, Z_4) (Z_1, Z_2, Z_3, Z_4)$.

Be triangular fuzzy intuitionistic numbers.

Now, by solving the problems (P3), (P2), (P4), (P1) and (P5) using the proposed method, we obtain the following optimal solutions of the crisp problems (P3), (P2), (P4), (P1) and (P5) respectively:

(P3) : $x_1 = 5.5, y_1 = 0$ and Maximize $Z_3 = 2$.

(P4) : $x_4 = 9.33, y_4 = 0, x_2 = 0, y_2 = 0$ and Maximize $Z_4 = 69.31$.

(P2) : Using the optimal solution of (P4), Maximize $Z_2 = 0.03$.

(P5) : $x_4 = 9.33, y_5 = 0, x_1 = 0, y_1 = 0$ and Maximize $Z_5 = 79.64$. 

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Using the optimal solution of $(P5)$

Maximize $Z_i = 0$.

Thus, the optimal IF solution of the given IFLFP problem is

$\tilde{x}_i = (0, 5.5, 9.33) (0, 5.5, 9.33)$,

$\tilde{x}_2 = (0, 0, 0) (0, 0, 0)$ and the maximum IF objective value is $\tilde{Z} = (0.33, 2, 69.31) (0, 2, 79.64)$.

REFERENCES


